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DEPARTMENT OF THE ARMY

CORPS OF ENGINEERS

BEACH EROSION BOARD  
OFFICE OF THE CHIEF OF ENGINEERS

STATISTICAL SIGNIFICANCE  
BEACH SAMPLING METHODS

TECHNICAL MEMORANDUM NO. 50

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# STATISTICAL SIGNIFICANCE OF BEACH SAMPLING METHODS



TECHNICAL MEMORANDUM NO. 50  
BEACH EROSION BOARD  
CORPS OF ENGINEERS

AUGUST 1954

## FOREWORD

Determination of beach material characteristics, and their variance across and along the beach, is important in many coastal studies -- in indicating the source and direction of movement of the littoral materials and, to some extent, the probable reaction of the beach to particular wave and tide conditions. Until very recently there has been no sound basis for planning a beach sampling operation, samples being taken in various ways and analyzed by various methods with little if any realization of the statistical implications involved. Studies have recently been underway which attempt to discover the best statistical model to be used in beach sampling (e.g., number and size of samples, depth of sampling, lateral and longitudinal spacing of samples), and the following is a report on one of these studies.

This report has been prepared by Dr. William C. Krumbein in pursuance of Contract DA-49-055-eng-35 with the Beach Erosion Board, which provides in part for research on programs and procedures for sampling littoral materials. Dr. Krumbein is Professor of Geology at Northwestern University.

Views and conclusions stated in this report are not necessarily those of the Beach Erosion Board.

This report is published under authority of Public Law 166, 79th Congress, approved July 31, 1945.



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STATISTICAL SIGNIFICANCE  
OF  
BEACH SAMPLING METHODS

by  
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INTRODUCTION

Characteristics of beach deposits are important in geological and engineering investigations of shore line processes. Almost every study of beaches includes collection of samples for determination of average particle size, mineral composition, and other attributes.

Most beach samples are collected with one or more of the following objectives in mind:

- (a) To obtain an estimate of the average coarseness or fineness of the beach material for descriptive or comparative purposes.
- (b) To obtain information on the characteristics of material required for beach nourishment or for the development of artificial beaches.
- (c) To determine the source rocks or source areas of the natural sand or gravel on the beach.
- (d) To determine whether there have been changes in the sources of the beach material within historic times due to man's modification of the beach area or its hinterland.

Inasmuch as the characteristics of beach material are in part dependent on physical processes operating in the beach area, the preceding specific objectives may be parts of a larger plan to relate the observed properties of the material to prevailing waves and currents, as well as to the distance that the material was moved from its original source area.

Methods of sampling beaches are largely empirical in that few studies have been made to test the effectiveness of different sampling plans. The purpose of this report is to review beach sampling methods in terms of some statistical principles, and to suggest tentative plans for improving the representativeness in the samples. In order to be self-contained the report includes some discussion of the statistical methods used. The results indicate that no radical revisions of current sampling

procedures are needed, but that recognition of explicit design elements in the sampling plan can result in greater reliability of the data for comparable expenditure of time and effort.

The writer is indebted to numerous individuals for suggestions and data used in this study. Messrs. R. O. Eaton, J. M. Caldwell, J. V. Hall, Jr., and T. Saville, Jr., of the Beach Erosion Board staff have been especially cooperative in discussing the problem and in placing beach information at the writer's disposal. Mr. Eaton has made many specific suggestions, and has read the manuscript critically for its engineering implications.

### REVIEW OF BEACH CHARACTERISTICS

Sampling programs to determine specific properties of the material being sampled are most effectively designed when the general characteristics of the material are known. In a statistical sense, each set of observations represents a sample drawn from some larger population. If the population is known to be relatively homogeneous, a set of random samples may supply sufficient data for estimating population characteristics. If the population shows natural subdivisions or gradients, the sampling plan may be adjusted to take such variations into account.

It is common knowledge that beach material characteristics change along and across beaches due to variations in the "natural treatments" imposed by waves and currents. Waves wash up to the berm, but seldom cross it. Seaward of the berm the sand is shifted about by wave action, but on the dry backshore, wind winnowing may selectively remove grains to the dunal belt. As a result of these differing processes, the beach as a whole displays a zonal arrangement, and is commonly divided into a backshore, foreshore, and nearshore bottom zone. The width and characteristics of the zones are functions in part of the amounts of beach material available, as well as of the larger environment of partially enclosed bay, open shore, or barrier system of which the beach is a part. Within this broader framework the sampling plan is thus part of a larger design that includes the physical setting of the beach.

The problem of designing a comprehensive sampling plan for such an interrelated system is beyond the scope of this preliminary study. Instead, attention will be confined to straight stretches of open beach as being more representative of conditions commonly sampled. Even here the problem is not simple. Cyclical changes occur on the beach seasonally and during storms. Low summer berms are topped by fall and winter storms, and new berms adjusted to prevailing wave conditions are developed. Even during a single season the changes which occur during storms in the neap tide range are noticeably different from those occurring in spring tide ranges. Likewise there is a difference in effects produced by storms occurring during rising or falling tides.

These cyclical changes affect beach widths, slopes, and textures of the beach material. Some beaches may be composed of sand in summer and pebbles in winter; in others (as some beaches along Lake Michigan) the beach may be composed of pebbles during quiet periods and of sand during storms. Some beaches are characterized by patches of gravel on a sand foundation; the position and size of the gravel patches may vary with storms and seasons. Any sampling plan which takes all these possibilities into account must be repetitive, and must extend from the highest berm crest to a water depth sufficient to include areas of significant amounts of sand movement.

The backshore beyond the highest berm is very seldom affected by wave action, and then only during severe storms or breakthroughs. From the sampling point of view the backshore and its belt of landward dunes may be considered a relatively unchanging part of the beach as far as hydraulic effects are concerned.

Inasmuch as emphasis in the present report is on engineering uses of beach data, the foreshore and nearshore bottom will receive most attention because they represent parts of the beach most continuously subject to hydraulic forces. Some aspects of the backshore are treated, however, to provide a basis for comparison with the more active portions of the beach.

#### BEACH POPULATIONS

It was mentioned that some knowledge of the populations from which samples are drawn is helpful in designing sampling plans. There are other advantages also, such as in the selection of statistical models for further analysis of the data, and in setting up confidence intervals for estimates of certain population parameters.

A considerable body of knowledge about some beach populations is available from the many geological and engineering studies that have been made. In part this knowledge is used by implication rather than explicitly. Some of the material is reviewed here to indicate that at least six kinds of distribution functions are commonly encountered in beach data.

In any small area on a large beach the average particle size, particle shape, mineral composition, moisture content, beach firmness, and other characteristics are relatively homogeneous, and closely spaced samples commonly show approximately normal distributions of these properties.

The distribution of average particle size in closely spaced beach sand samples tends to be normal, although the distribution of grain sizes in any single sample is skewed. The individual samples may be "normalized" by a log transformation, as is commonly done graphically by plotting the size data on logarithmic paper.

Some beach characteristics have a directional or angular sense, such as foreshore slopes, orientations of grain axes, and other attributes. These commonly show symmetrical distributions which can be approximated by the normal curve. In theory, however, these angular distributions are circular normal.

Some beach data show highly skewed frequency distributions which cannot be normalized by a log transformation. These are Gamma distributions, and they occur when the rarer chemical constituents of sedimentary deposits are studied, for example. In some cases the distribution of bedding thicknesses follows a Gamma distribution.

The preceding kinds of distributions apply to continuous data. In contrast to these are discrete distributions, where the variables takes on only integral values. These distributions are encountered when mineral grains in sands are counted, such as in the analysis of heavy minerals. The distributions are developed by taking subsamples of fixed size (say 25 to 50 grains), and counting the number of specific occurrences per subsample. If the mineral counted is fairly abundant, each subsample will have one or more grains, whereas if the mineral is rare, some of the subsamples may not contain any of the grains. These conditions give rise to two kinds of distributions. The binominal distribution, relatively symmetrical, is found when the minerals are abundant, and the Poisson distribution occurs for very rare minerals. Poisson distributions are commonly highly skewed.

Table 1 summarizes the six kinds of distributions, and includes several examples of each type. The table serves as background material for later mention of certain beach populations. Figure 1 shows an example of each kind of distribution for illustrative purposes.

#### BEACH ZONES

It was mentioned that beaches may be roughly divided into the backshore, the foreshore, and the nearshore bottom, as indicated in Figure 2. In a statistical sense these constitute three or more "sampling strata", each of which has its own characteristic populations. The strata may be considered as separate units, they may be further subdivided, or they may be considered as parts of a "superpopulation" which comprises the beach as a whole. In many instances there are continuous gradations between the zones, so that the population as a whole may have gradients from one zone to another.

An easily visualized example of populations is illustrated by the moisture content of beach sand. In the nearshore bottom all the pores among the sand grains are filled with water and the moisture content is of the order of 35 to 40 percent. On the foreshore the samples close to the water line may have 20 to 25 percent moisture, and as samples are collected higher on the foreshore this percentage rapidly drops to between 5 and 10 percent. On the crest of the berm there may be only 1 or 2 percent moisture, and the somewhat lower elevation of the backshore may develop slightly higher moisture values.

TABLE 1

FREQUENCY DISTRIBUTION OF BEACH DATA\*

1. NORMAL, OR NORMAL APPROXIMATION
  - Average particle size
  - Particle roundness and sphericity in some samples
  - Moisture content of beach sand
  - Beach firmness
  - Porosity of beach sand
  - Orientation of particle axes when range is small
  - Beach slopes expressed in angles
  - Percentages of abundant minerals in sand
2. LOG NORMAL
  - Particle size distribution in single samples
  - Permeability of beach sands
  - Thickness of bedding (?)
3. CIRCULAR NORMAL
  - Orientation of particle axes when range is large
  - Orientation of cross-bedding in beach deposits (?)
  - Distribution of angle of approach of waves
4. BINOMIAL
  - Abundant minerals in sand or of rock types in gravel  
(when data are expressed as number of particles occurring  
in subsamples of fixed size.)
  - Number of frosted grains in some dune sands (in subsamples of  
fixed size).
5. POISSON
  - Rare minerals in sand (or of rock types in gravel), when data  
are expressed as number of particles per subsample of fixed  
size.
6. GAMMA DISTRIBUTIONS
  - Thickness of bedding (?)
  - Percentage of rarer minerals in sand
  - Particle roundness and sphericity in some samples

\* Exceptions to this classification occur. Note also that percentages are usually binomial or Poisson basically, but as continuous variables they apparently may be treated as normal approximations or Gamma distributions.

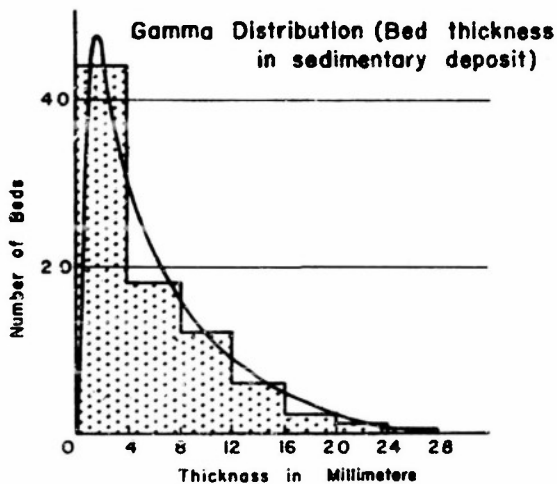
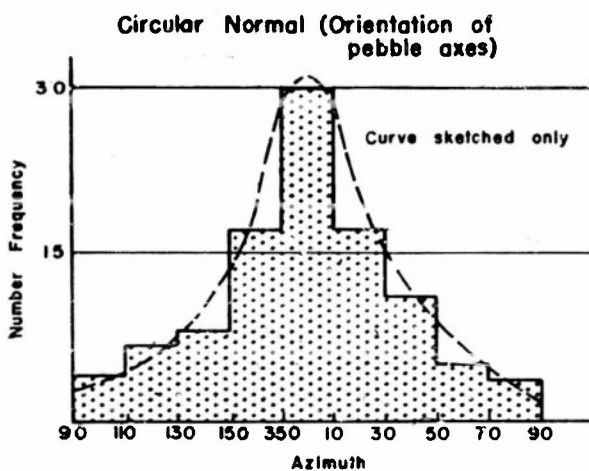
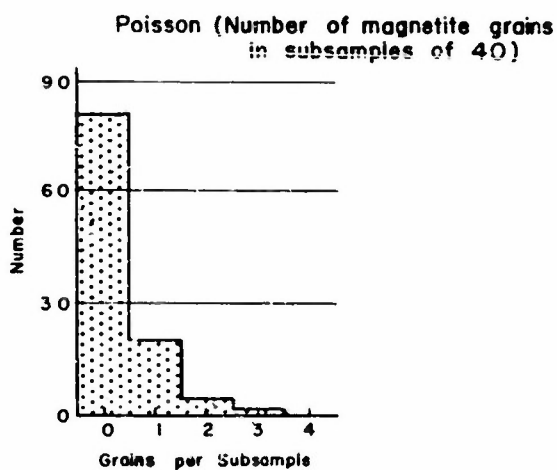
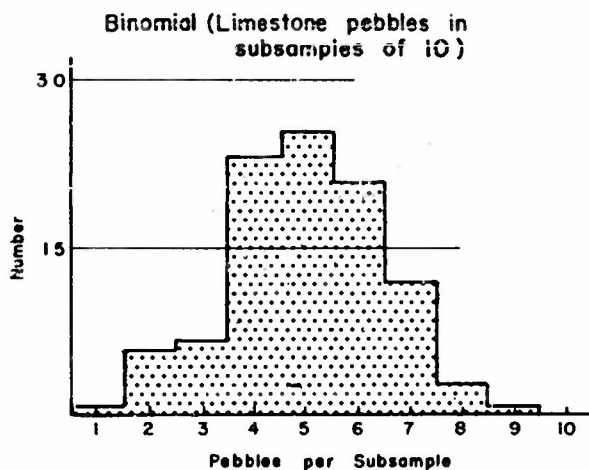
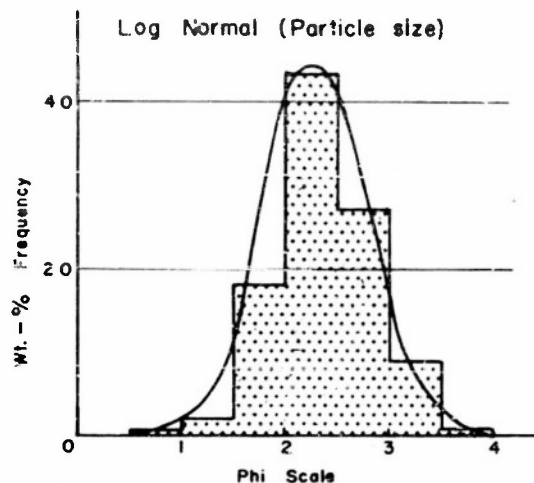
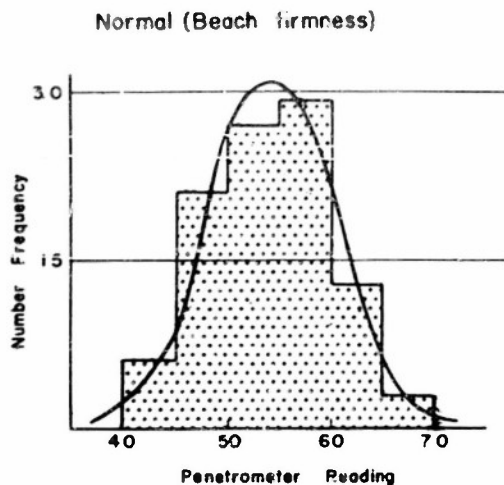


FIGURE 1 - DISTRIBUTION FUNCTIONS ENCOUNTERED IN BEACH SAMPLING

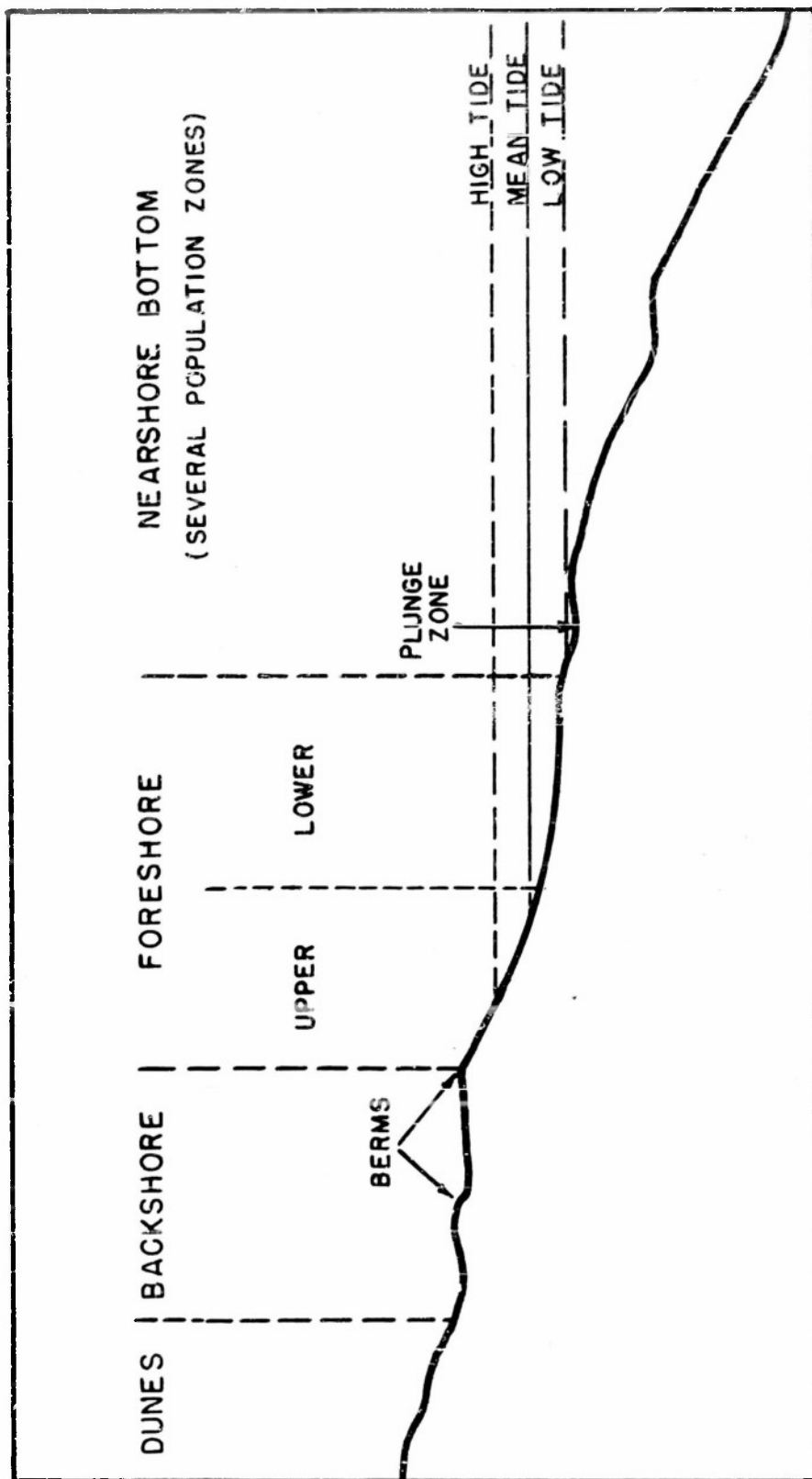


FIGURE 2 BEACH ZONES (SAMPLING STRATA)



In treating the moisture data, one may consider that the entire beach has one superpopulation of moisture content, with a sharp gradient from the water line landward. On the other hand, one may consider the nearshore bottom and the backshore as two distinct populations with a transition zone between. In fact, one may set up several parallel population bands across the beach, depending upon the limitations he wishes to impose upon any single "population."

Although the changes may not be so obvious to the eye, similar variations occur in particle size, mineral composition, beach firmness, and other attributes from zone to zone. The degree of variability itself varies with different properties, so that the beach as a whole may be considered as a complex of populations of various attributes, each with its own characteristics and rates of change.

In part, the sampling plan depends upon the population model adopted. In many instances it is preferable to treat each zone separately, although in some studies a broad over-all value for the beach as a whole may be adequate.

#### CLASSIFICATION OF SAMPLING METHODS

Methods of beach sampling have in large part developed from experience, and only in relatively recent years has special attention been paid to the statistical problem of obtaining the best kind of examples for a given study. During the past two decades statisticians have developed sampling theory to an advanced level in many fields (such as agricultural sampling, opinion sampling, etc.). The body of sampling theory available applies equally well in principle to other fields, and some of the definitions and concepts are of direct value to beach sampling.

A distinction should be made between purposive selection and random sampling. In purposive selection the samples are restricted to deposits or places considered by the worker to be especially typical of the conditions being sampled. A random sample is one in which some elements of random selection has been introduced to avoid personal bias, and it is in connection with the latter kind of samples that sampling theory is developed. Cochran (1953) develops this point.

The requirement that an element of randomness be introduced into sampling does not mean that the sample is to be taken blindfolded, as it were, without regard for the characteristics being sampled. Rather, the requirement is that some process of randomization be applied to the procedure of collecting the sample. This randomization process may take different forms, depending upon the kinds of samples being collected. The distinction is brought out in the following definitions of sampling procedures. The formal definitions are given by Cochran (1953); they are paraphrased here to apply more directly to beach sampling, either along a profile or over an area.

In sampling sedimentary deposits, the unit of sampling is a small volume of the deposit rather than an individual grain, although in some

studies the grains themselves may be sampled. In the following treatment it will be assumed that the beach sample is a cylinder of sand about 3 inches in diameter and 2 inches deep. Four kinds of sampling procedures are available, which will be illustrated first by samples collected along a profile line across the beach.

1. Simple random sampling is a method of selecting a sample of given size such that every one of the possible samples of this size along the line has an equal chance of being selected.

2. Stratified sampling involves dividing the sampling line into L non-overlapping segments (strata) which together constitute the entire length of the line. A set of L samples, representing one sample collected at random from each segment, is a set of stratified random samples.

3. Systematic sampling is performed by taking a sample at random from an arbitrarily defined segment of length k, and thereafter selecting a sample from the same relative position in all remaining segments of length k along the original sampling line.

4. Stratified systematic sampling is performed by first dividing the line into L strata as under (2). A sample is collected at random in the first stratum, and thereafter a sample is taken from this same relative position in each of the remaining strata.

Figure 3 comprises plan views of a beach profile 1,000 feet long extending from the inner edge of the backshore to the 30-foot depth in the nearshore bottom. The four lines in the figure represent the same profile sampled in four different manners. In profile A five samples are collected at random. One way of selecting the samples is to consider the profile as comprising 4,000 consecutive sampling positions, each 3 inches long, numbered from 0001 to 4,000. A table of random numbers (such as those in Dixon and Massey, p. 290) is entered at some convenient point, and the first five 4-digit numbers are listed, ignoring numbers larger than 4,000, and omitting duplicates. Say the numbers are 1126, 2724, 0249, 1196, and 3253. Then a sample is taken from the 249th, 1126th, 1196th, 2724th, and 3253rd 3-inch segments, representing distances of 62.25, 281.50, 299.00, 681.00, and 813.25 feet from the landward end of the profile. These constitute a set of simple random samples, with some qualifications developed later.

In profile B the beach is first stratified into backshore, upper and lower foreshores, and say two nearshore bottom zones. These are natural strata suggested by past experience on the beach, and they constitute the L segments of the definition. Each stratum is now considered as a sampling segment, and a separate random number is drawn for each stratum from a range of values equal to the number of possible 3-inch sample spacings along the line. The five resulting samples constitute a set of stratified random samples inasmuch as each stratum was sampled in one random position. For strata of different lengths it may in some cases be desirable to take a number of samples from each stratum proportional to the length of the stratum.

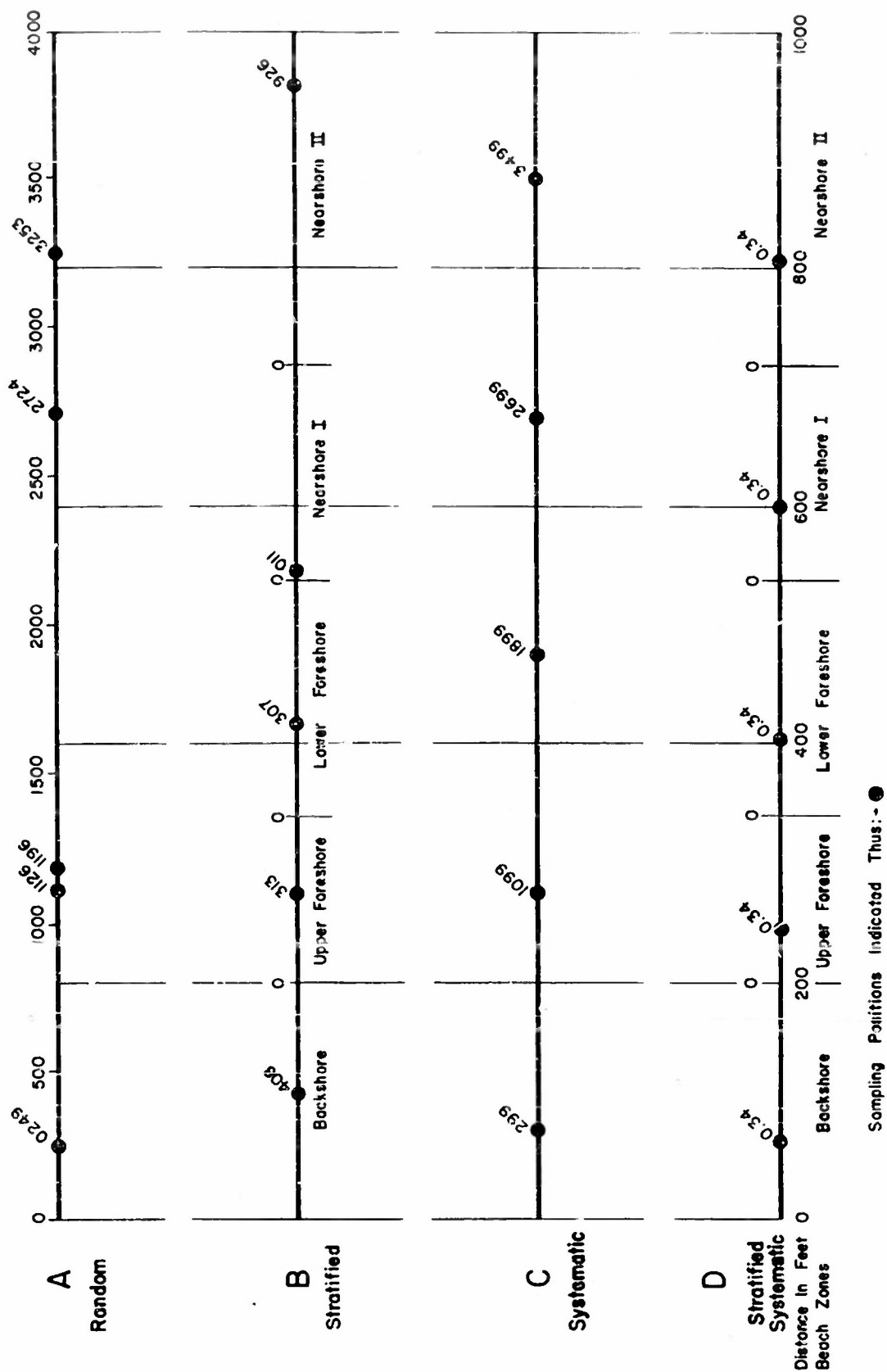


FIGURE 3 - SAMPLING PLANS (BEACH PROFILE)

In profile C the sampling line is considered as having 4,000 possible sampling sites as in A, but it is now arbitrarily decided to consider the profile as being composed of five sampling segments of equal length. These are the k groups, each of which has 800 of the original 4,000 3-inch sampling elements. A single number between 001 and 800 is selected at random, say 299, and the first k group is sampled at the 299th position. This same element is now sampled in the remaining k segments, i.e., numbers 1099, 1899, 2699, and 3499 on the original numbering scheme. The five samples constitute a set of systematic samples along the profile.

In profile D stratified systematic sampling is used. When the strata are of equal length it is only necessary to select a number at random within the range of sampling positions, and to repeat the sampling at the same point in each stratum. In the present case the strata are of unequal length, and it was decided to select a number between 00 and 99 to represent a proportional distance in the stratum. The random number table yielded 34, which was used as 0.34 to locate a sample at that proportionate distance from the initial point in each stratum.

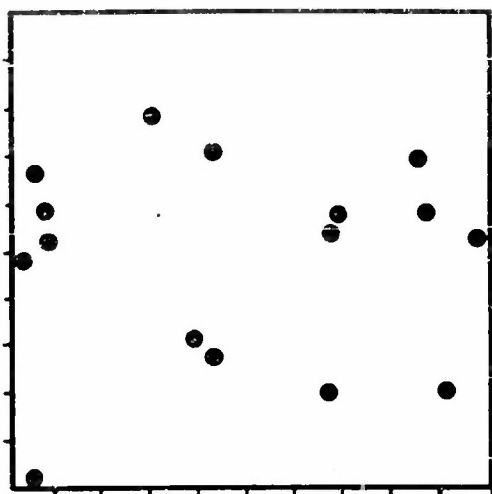
In considering these four alternative methods for sampling along a profile, the question may be raised whether any one of the methods is superior to the others. If the beach population along the profile is homogeneous, simple random sampling provides a satisfactory estimate of the beach population parameters. If the beach population varies markedly along the profile, simple random samples may in some instances give a poor estimate, because the random numbers may all fall into a single natural beach zone or stratum. Inasmuch as experience amply demonstrates the occurrence of these natural zones on the beach, stratified sampling seems to be preferable. Whether samples taken at random in each stratum are preferable to samples systematized in each stratum probably cannot be answered without some knowledge of the population gradients involved. Cochran (1953) discusses some of the factors involved in populations with gradients, with cyclical changes, etc.

When a beach area, as opposed to a profile line, is considered, the several types of sampling become applications of plane sampling, also discussed by Cochran (1953, p. 183) and developed in greater detail by Quenouille (1949). Suppose an area 100 x 100 feet on the backshore is to be studied for its characteristics, and it is decided to take 16 samples from the area. If each sample is to occupy a 3-inch circle, there are some 160,000 possible sampling positions available.

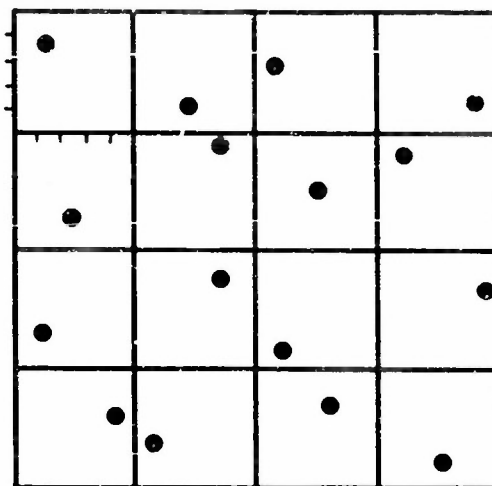
The 100-foot edges of the large square are numbered from 001 to 400 in sequence, suggested in Figure 4A. Sixteen pairs of 3-digit numbers in this range (omitting duplicate pairs) are drawn from a random number table, and the samples are taken from the corresponding positions, using the paired numbers as x- and y- coordinates. This yields a set of 16 random samples from the beach area.

In Figure 4B the 100-foot square is divided into 16 smaller squares by driving stakes at 25-foot intervals over the larger square. Each of the 25-foot squares is numbered from 00 to 99 along the edges. By

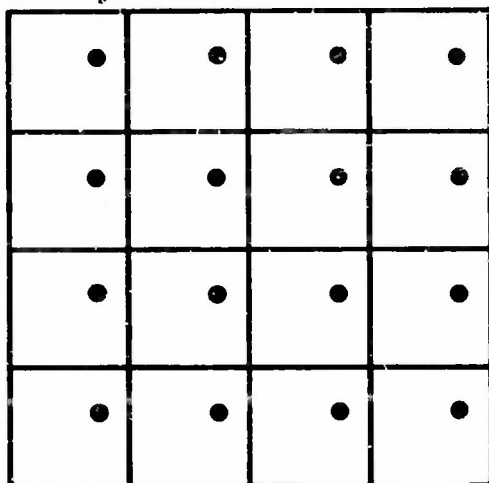
A - Random



B - Stratified



C - Systematic



D - Stratified Systematic

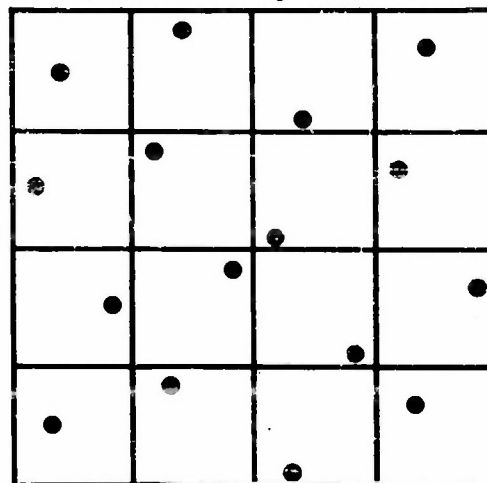


FIGURE 4 - SAMPLING PLANS (BEACH AREA)

drawing 16 pairs of numbers in this range and collecting samples at the corresponding points, a set of stratified random samples is obtained. In this instance the subdivision of the 100-foot square into smaller squares is a process of stratification in two dimensions.

In Figure 4C a sample is randomized in the upper left 25-foot square by drawing a pair of random numbers between 00 and 99. This same position is now sampled in the remaining 15 squares. The 16 samples constitute a set of aligned systematic samples, corresponding to stratified systematic samples along a line. When samples are taken directly at the stakes, which has been a common procedure, the samples have the same kind of pattern, and the randomizing element is preserved by randomizing the position on the backshore of one corner of the major 100-foot square.

Another form of systematization, yielding unaligned systematic samples, is defined by Quenouille (1949) and also discussed by Cochran (p. 183). This is shown in Figure 4D. The position of a sample in the upper left 25-foot square is randomized by selecting a pair of numbers between 00 and 99, representing the vertical and horizontal coordinates. This is followed by randomizing the horizontal coordinates in the remaining squares of the upper row, and randomizing the vertical coordinates of the remaining squares in the first column. The resulting pattern of sampling points is systematic in the sense that the distance, horizontal or vertical between samples in successive strata is the same.

Quenouille considered the relative efficiency of these several sampling plans, as well as several others. In general it may be said that if the backshore area is homogeneous, any of the methods will give fair estimates. If a gradient is present in the population, however, it seems probable that the unaligned systematic samples will detect it most effectively. This may be seen by comparing Figures 4C and 4D. In 4C a gradient from left to right, for example, would be picked up at the same four points in each row. In 4D, on the other hand, the different positions of the samples in each row would result in 16 intersections of the gradient.

The preceding discussion gave equal weight to each 3-inch circle in the sampled area, which would require rather accurate location with transit and tape. For practical purposes one may decide to randomize to the nearest square foot, taking the sample from the edge or center of the designated square. For most beach studies it is probably adequate to prepare a drawing or map of the sampling design in the office, randomize the sample positions on the map, and collect them in the field to the nearest conveniently measurable unit.

In some detailed beach studies, and in connection with some analysis of variance designs, it is desirable to have more than one sample in a grid cell. Multiple samples are obtained by repeated application of the same randomizing element for succeeding samples. In similar fashion, it may be desirable to collect more than one sample from each beach zone along a profile. This may also be accomplished by successive randomization of the sample positions. An alternative is to take a number of



samples from each zone proportional to the zone width. This gives rise to a "representative sample" in that each zone (or stratum) is represented according to its size.

Multilevel sampling has not been mentioned explicitly, although in some respects it is implied in the compromises with randomization made in an earlier paragraph. Thus, adoption of 1 square foot as a unit sampling area, with a 3-inch sample taken from the center, is a form of multilevel sampling. This could be made explicit by selecting the square foot at random, and then randomizing the position of the sample within the square foot. Whether these added refinements are desirable in routine engineering studies may be questioned, although it is appropriate that the question be examined academically.

In the more usual form of multilevel sampling, one might select several 100-foot squares along the backshore and divide each into 25-foot squares. Two or more of the 25-foot squares may be picked at random from each 100-foot square, and within each selected 25-foot square two or more samples may be taken at random. This would produce a set of samples in a three-level hierarchy, and from the data, the variance contributions of each level of sampling may be computed. Inasmuch as the method may be used to test the effects of sample spacing, an example will be given in a later section.

Cluster Sampling is a process of collecting a group of closely spaced samples at each major sampling point. Thus, if multiple samples are to be collected from each beach zone along a profile, a stake could be driven in the beach at the randomized sampling points (as in Figure 3B), and two or more samples could be collected around each stake. One way of doing this is to draw a circle about 2 feet in diameter around the stake, and divide the circle into say 16 equal segments. By drawing numbers from 01 to 16 (omitting duplicates) from a random number table, the samples could be taken from the corresponding circle segments, say 1 foot from the stake. To avoid finite population corrections (Cochran, 1953) the number of segments should be fairly large compared to the number of samples in a cluster.

Cluster sampling as described here involves randomization of the main sampling point and of individual samples in the cluster. The sand sample itself is a type of cluster sample in which the randomizing process is applied to the total sample, but not to the individual sand grains. This type of sample is sometimes referred to as a "complex random sample".

The many implications of the several sampling plans treated here are not further discussed in this report, although Cochran (1953) develops the corresponding statistical theory for each kind of sample. In part the selection of the plan depends upon costs, and upon whether the main objective is to obtain a population estimate or to study relationships within the populations more analytically. Beach sampling includes both aspects, and Cochran's book is recommended as a basic text for further analysis of the sample data.

## ANALYSIS OF BEACH SAMPLES

The preceding section showed that general principles of sampling may be applied to beaches and that the application of these principles provides a number of alternative ways for laying out samples along profiles or on a grid. The question of the best choice of method depends in part on the objectives of the study, and this in turn depends upon the use to be made of the samples. In this section some of the kinds of analyses performed on beach samples are discussed, to pave the way for more detailed consideration of the selection of specific sampling methods.

Samples from beaches are analyzed for various attributes. The most common is particle size analysis, which may be conducted with sieves, with a settling tube, or by measuring grains under the microscope. There is considerable discussion of the relative merits of these methods in the literature, inasmuch as each involves a different basic definition of "size". For practical purposes it is perhaps sufficient that all size data used in any one study be obtained by the same process, so that the mean sizes obtained represent the same operationally-defined variable. Only when sets of data involving different methods are compared is it necessary to show that one method is equivalent to another. Various statistical tests for showing such equivalence or its lack are available.

In addition to variations in methods of analysis, there is also considerable variation in methods of expressing mean particle size. Perhaps the most commonly used is the median diameter in millimeters, which represents the size of the middlemost grain in the weight frequency distribution. Earlier practice had been to use the arithmetic mean diameter in millimeters, graphically determined as the "equivalent grade". Recognition that many beach deposits approach log normalcy in their size distribution led to the development of the logarithmic mean (phi mean) and its antilog, the geometric mean diameter in millimeters. Use of the log median (phi median) is also common with some workers.

In similar manner, the degree of spread of the grains about the mean size, is expressed in various ways. Those most commonly used are the grading factor, a variant of the mean deviation; the sorting coefficient based on quartile ratio; and a log standard deviation (phi standard deviation) based on the log transformed data.

The relative merits of the methods of expressing particle size are not of major concern here, except to point out that in general the more suitable measures are those in which the mean values tend to approach a normal distribution, and in which the measures of spread tend to approach a Chi-square distribution. Statistical analysis of the sample data is facilitated under these conditions. Fortunately, the mean grain size in all methods of analysis is based upon a large number (or weight) of grains, so that most of the average values used probably tend toward symmetrical distributions. This point needs some verification; perhaps more important, the sample distribution of the various sorting measures needs critical study.



In any event, particle size analysis is usually expressed as an average diameter, a measure of spread, and in some instances a measure of skewness. These are the data that are generally available for further statistical analysis.

The fact that each beach sample provides an estimate of average diameter, spread, and skewness, raises certain complexities in further statistical analysis of a group of samples. It was mentioned that beach sand samples are a kind of cluster sample because each sample contains a large number of grains. Inasmuch as the randomizing process is applied to the cluster and not to the individual grains, the latter are not independent in a statistical sense. Moreover, the mean diameter and other measures from each sample are commonly based on weight percentage frequency, whereas further analysis is based on number frequency. Finally, the sample average is commonly expressed as a median diameter, whereas subsequent analysis involves the arithmetic mean of the medians. The assistance of mathematical statisticians will be required to sort out the variances contributed by each of these procedures.

In addition to particle size analysis, beach samples are commonly studied for their mineral composition. Most beach sands are composed predominantly of quartz and other light-colored minerals, with a small percentage of darker minerals. These darker minerals have a greater specific gravity and are separated from the sand with heavy liquids. The heavy minerals are important in identifying the source rocks and source areas of the sand. Common practice is to count several hundred to a thousand grains, noting the abundance of the several species present. The results are commonly expressed in number percentage terms. As was pointed out previously, the distribution of minerals in sediments tends to be binomial for the more abundant ones and Poisson for the rarer ones.

In some special studies the shape of the grains is also studied. Shape may be expressed in terms of the sphericity (operationally defined as the cube root of the ratio of particle volume to volume of a sphere circumscribed about the particle), and as the roundness (operationally defined as the ratio of average radii of corners and edges to radius of circle inscribed in maximum projection plane of the particle). Various short methods for estimating these properties are available.

Particle orientation (compass direction of longest particle axis in space, for example) is not extensively studied for engineering purposes, although some methods are available for pebbles and large sand grains.

Other kinds of observations which may be made on beach deposits include study of beach bedding, beach firmness, beach slope, and such special features as beach cusps, etc.

From a statistical point of view the size of a sample should be adjusted to the beach characteristics being studied. In practice, however, most beach samples are of such size that abundant grains are available for any kind of analysis. In special studies it may be found that sample

spacing is influenced by the data of main interest. Decisions on this point must in part await additional knowledge of the population gradients of particle size, particle shape, mineral content, and the like.

Perhaps the most common problems that arise in connection with the several objectives listed in the introduction are concerned with estimates of the over-all characteristics of the beach deposit; with evaluation of changes in texture or composition along and across beaches; or with comparison of deposits on the same beach during different seasons. Inasmuch as average particle size is one of the main considerations in such comparisons, it is used here as the main illustration, although the general approach is also applicable to questions of source areas for beaches.

#### COMPARISON OF SETS OF BEACH DATA

In proper perspective beach sampling is a means toward an end. The objective of the study is to learn something about a beach, but the kinds of tests that can be made on the data, and the convenience of computation, are influenced by the number and kinds of samples available.

As a foundation for the closing section of this report on recommended sampling procedures, the beach problems mentioned at the close of the preceding section are treated here. Several standard statistical methods are introduced to indicate ways in which the problems may be approached.

A basic problem that arises in beach studies is the evaluation of seasonal changes on beaches. Suppose samples are collected on the same beach throughout a year and it is desired to test whether there is a significant difference in average particle size over the seasons. Handin (1951) furnishes tables of data which may be used as illustrations. Several samples were collected each month from a number of beaches. When several such groups of samples are available, it is possible to apply analysis of variance to the data, and to test all twelve months simultaneously.

Certain assumptions underlie applications of analysis of variance, as discussed by Eisenhart (1947), and Dixon and Massey (1951). Some preliminary tests applicable to the data are mentioned later.

In its simplest form analysis of variance compares variances between and within groups of data. If the ratio of the between-group variance to the within-group variance exceeds a selected critical value, the difference between the groups may be considered statistically significant. Dixon and Massey (1951) give an introduction to analysis of variance, and describe the method of the following example on p. 121.

Table 2 shows the data arranged in month columns. The number of observations per month ranges from three to five. The design used is a single factor basic form, and the form of the analysis is shown at the bottom of Table 2.

Although the methods of computation for this model are given in detail in Dixon and Massey, they are included here for readers unfamiliar with the technique. The computations are encumbered by extra operations because of unequal numbers of items in each monthly group. The first step is to

square all the items in the table,  $(324)^2 + (423)^2 + (341)^2 + \dots + (335)^2 + (341)^2 + (466)^2 = 6,425,258$ . This may be called  $S_1$ . The second step is to square each group total and divide it by the number of items in the group, and then add the quotients:  $(1810)^2/5 + (1910)^2/4 + \dots + (1142)^2/3 = 6,338,409$ . Call this  $S_2$ . The last step in squaring is to square the grand total and divide it by the total number of items in the table:  $(16944)^2/46 = 6,241,286$ . Call this  $S_3$ .

TABLE 2

ARITHMETIC MEAN DIAMETER OF BEACH SAND COLLECTED FROM SAME BEACH DURING EACH MONTH (Data from Handin, 1951, Station 56, p. 108. Values expressed as microns)

Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.
324	505	355	348	291	418	314	350	380	392	274	335
423	525	370	388	345	303	319	288	383	410	290	341
341	460	430	322	311	458	296	388	476	382	347	466
375	420	361	432		232	318	304	381			
347			426								
TOTALS	1810	1910	1516	1916	947	1411	1247	1330	1620	1184	1142
$\bar{X} =$	362	478	379	383	316	353	312	333	405	395	381

ANALYSIS OF VARIANCE

<u>Source</u>	<u>Sum of Squares</u>	<u>Degrees of Freedom</u>	<u>Mean Square</u>	<u>F</u>
Between Months	97123	11	8829	3.46**
Within months	86849	34	2554	
Total	183972	45		

The sums of squares (SS) in Table 2 are obtained as follows: The total SS is found by subtracting  $S_3$  from  $S_1$ :  $6,425,258 - 6,241,286 = 183,972$ . The between-group SS is found by subtracting  $S_3$  from  $S_2$ :  $6,338,409 - 6,241,286 = 97,123$ . Finally, the within-group SS is obtained by difference,  $183,972 - 97,123 = 86,849$ .

The degrees of freedom associated with the groups is one less than the number of groups,  $12-1 = 11$ . The total degrees of freedom are one less than the total number of items,  $46 - 1 = 45$ . The within-group sum of squares is the difference between these:  $45 - 11 = 34$ .

The mean squares are found by dividing the sums of squares by the corresponding degrees of freedom to obtain the values shown in Table 2.

The computational process thus separates the total variability into two sources of variability. The statistical test is to see whether the between-group mean square is significantly larger than the within-group mean square. In making this test one has to decide beforehand on the level of significance at which he is willing to judge the results. In the present instance the statistical hypothesis is set up that there is no significant difference between the group means at the conventional 5% significance level. Selection of the 5% level implies that one is taking the risk of falsely rejecting the hypothesis once in 20 times on the average. The risk of making this false decision may be reduced by accepting a 1% significance level, but in that case there is a greater risk of not rejecting the hypothesis if the means really are different.

In the present experiment the 5% level is chosen. The ratio  $F$  of the between-group to the within-group mean square is computed and found to be 3.46. The critical value of the  $F$  ratio is looked up in a table of the  $F$  distribution (Dixon and Massey, p. 310) for 11 and 34 degrees of freedom. By interpolation it is found to be about 2.08. The observed value is larger than this, and hence is statistically significant at the 5% level. The hypothesis of no significant differences among the group means is accordingly rejected.

One may also check the  $F$  value for the same degrees of freedom at the 1% level. The table on p. 312 of Dixon and Massey yields the value 2.82, approximately. This is also smaller than 3.46, so that the ratio is significant at the 1% level. This is shown in Table 2 by adding two stars after the figure.

One may conclude from this experiment that there is a highly significant difference between the monthly means of average particle size on this beach. This implies a strong seasonal change, but it does not identify the specific months in which the changes occurred. Methods of contrasts are available for such tests, but a simpler method in the present instance is to combine the original data into two main groups, say from April to September and from October to March inclusive, and use the  $t$ -test on the two groups (Dixon and Massey, 1951, p. 102). When this test is run on these data, it is found that a significant gross seasonal difference is also present.

The one factor analysis of variance form may be used in any study involving  $k$  groups of  $n$  items each. These may represent samples collected in different months on the same beach; different profiles sampled at the same time along a given beach; samples from different beaches taken at the same time; or groups of samples from different zones on a given beach.

A somewhat more versatile analysis of variance design of considerable value in beach studies is the row-column (two factor) form. This permits simultaneous comparison of two factors, such as changes along and across beaches; different parts of the same beach sampled at different times; different beaches sampled at different times; and other combinations.

For this design samples may be collected on grids, and the following illustration, taken from an earlier paper by the writer (1953) illustrates a study of population homogeneity on a beach at Wilmette, Illinois. The data are shown in Table 3. The upper three rows of samples were taken from the backshore, and the lower row was taken from the foreshore.

TABLE 3

GRID OF VARIATION IN LOG MEAN PARTICLE SIZE; DATA  
EXPRESSED AS PHI MEANS OF PARTICLE SIZE DISTRIBUTION

ALONG BEACH					
Across Beach	Backshore	2.15	2.06	2.10	1.88
	Backshore	2.10	2.06	1.86	1.95
	Backshore	1.96	1.90	2.07	1.87
	Foreshore	0.27	0.52	0.97	1.18
ANALYSIS OF VARIANCE					
Source	Sum of Squares	Degrees of Freedom	Mean Square	F	
Between columns (along beach)	0.0766	4	0.0192	< 1	NS
Between rows (across beach)	5.0528	3	1.6843	26.48	**
Residual	0.7634	12	0.0636		
Total	5.8928	19			

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The problem here was to compare average particle size along and across the beach on a sampling grid that straddles the berm line. The statistical hypothesis to be tested is that there are no column effects (along beach) and no row effects (across beach) at the 5% significance level. The design and computations are described by Dixon and Massey (p. 127). The total variability is separated into three parts, the first associated with the column effect, the second with the row effect, and the third with a "residual" variance which represents the error terms. The two F ratios are obtained by dividing the column and row mean squares respectively by the residual mean square.

The analysis shows that there is a highly significant row effect, but no significant column effect (as indicated by the letters NS). This means in essence that the backshore population may be considered separate from the foreshore population, whereas in both these populations there is no significant change in means along the length of the grid (about 200 feet).

When prominent zone changes occur, as in this example, the use of the gross average to characterize beach particle size is generally less effective than the use of two means, one for the backshore and one for



the foreshore. In the original paper the separation of the populations was affected by a second analysis of variance.

On the basis of the second test it was inferred that the three upper rows constitute one population and the lowest row another. Hence, in estimating population means, the values in the upper three rows may be separately averaged to obtain a phi mean of 1.99, corresponding to a geometric mean diameter of 0.25 millimeter. The foreshore phi mean, on the other hand, is 0.83, corresponding to a geometric mean diameter of 0.56 millimeter. Thus, the foreshore particle population is more than twice as coarse as the backshore. A gross average of all values in the table is 1.70 for the phi mean, corresponding to 0.31 millimeter for the geometric mean. This value is not closely representative of any sample in the grid.

A suggestion from this example is that if estimates are to be made of average particle size on given parts of beaches, the sampling plan should involve stratification. By including in each group only samples from a homogeneous population, the mean values will be more characteristic of the zone, and the variance among the means will generally be smaller.

The examples presented here represent several basic statistical models for comparing groups of data. More advanced analysis of variance designs are available for problems of wider scope. For example, use of a three-factor analysis of variance design would permit simultaneous evaluation of changes along a beach, across a beach, and seasonally. An example of this type of design is given by Krumbein (1953). Dixon and Massey (1951) is highly recommended as an introductory text, both for analysis of variance and for definitions of terms used in this paper. Hoel (1947) provides a somewhat more mathematical introduction to statistics.

The analyses of variance designs included here are used mainly for testing hypotheses about population means on a single beach. Yet any one beach is only a sample from the much larger population of all beaches. If interest shifts from consideration of fixed relations among means to consideration of random deviations in the characteristics of any one beach from the mean value of these characteristics in the much larger population of all beaches, a model appropriate for estimation of variance components may be used. An excellent treatment of these two kinds of analysis of variance models is given by Eisenhart (1947).

#### SAMPLING PROCEDURES

In preceding sections no question was raised about sample size except to point out that usual practice affords samples of adequate size for most purposes. Analysis of variance may be used to study this problem more critically, and elsewhere the writer (1953, 1954) has shown that the diameter of the sample is of less importance than the depth of penetration for both sand and gravel beaches. The experiments cited apply to a sand beach at Wilmette and a gravel beach in Evanston. More recently the experiment was repeated on the backshore at Ocean Beach, Maryland, with the same results. This experiment is shown in Table 4. On the basis of

these experiments it seems relatively safe to infer that a sand sample from about 1.5 to 3 inches in diameter is satisfactory for particle size analysis.

TABLE 1

SAMPLE -SIZE STUDY OF BEACH SAND FROM BACKSHORE,  
OCEAN BEACH, MARYLAND (DATA EXPRESSED AS PHI  
MEAN OF PARTICLE SIZE DISTRIBUTION.)

Depth of Sample	Diameter of Sample			
	1.5"	3"	6"	12"
3"	2.05	2.06	2.05	2.05
6"	1.97	1.98	1.97	1.98
9"	1.87	1.88	1.86	1.86
12"	1.69	1.69	1.67	1.66

ANALYSIS OF VARIANCE

<u>Source</u>	<u>Sum of Squares</u>	<u>Degrees of Freedom</u>	<u>Mean Square</u>	<u>F</u>
Between columns (sample area)	0.0006	3	0.00020	3.33 NS
Between rows (sample depth)	.3170	3	.10570	176.2 **
Residual	.0005	9		
Total	<u>0.3181</u>	<u>15</u>		

Common practice is to emphasize sampling of the upper few inches of the beach, but there seems to be little organized information on the relative differences that would show up seasonally or along a beach as a function of the depth of sample penetration.

That the problem of sample depth is important is recognized by most workers. It is not unusual to have segments of the beach up to several feet thick cut out or filled in near the berm during storm cycles. If samples in one zone penetrate the surface of a newly-deposited layer, whereas in another zone the samples penetrate a scoured layer normally several feet beneath the surface, there may be a real question whether the samples are strictly comparable in terms of operating processes. There is no simple way of avoiding such sampling complexities, although some evidence on the occurrence of marked changes could be had by running elevations on each profile at successive times of sampling to determine whether any serious scour had occurred.

There is some justification in the practice of sampling the upper few inches despite these complexities. From an engineering point of view

an important consideration is to determine the beach characteristics in the zone of active hydraulic forces, and if scour and fill occur, that very fact may add to the variability of the beach surface through time. Storm-cycle or seasonal variability may be important in groin design. for example, in that changes in average particle size on the foreshore through the seasons will be accompanied by changes in foreshore slope. Some knowledge of such changes is valuable in designing groins adjusted to the expected range in foreshore slope, entirely aside from the specific processes which may produce the changes.

Space limitations prevent complete discussion of sampling devices, although the subject is important in its influence on the quality and quantity of sample obtained. Samples on the exposed beach may be collected by digging a hole to a given depth, mixing the material extracted, and taking a pint or quart sample by some suitable subsampling method. Perhaps more common is the practice of rotating a pint or half-pint ice cream carton into the sand.. In some instances the top half-inch of surface sand is first scraped off. Normally a single sample is taken at each sampling point.

Underwater samplers appear to be of three main kinds. The sampler collects a single sample either by penetration into the bottom, by scraping up a surficial layer, or by "grabbing" a sample in a snapping motion. The relative efficiency of samplers has been looked into, and more could be done along this line with designed experiments. For present purposes it will be assumed that a sampler is used which provides samples of about the same size and depth of penetration as are collected on the above-water part of the beach.

Most underwater samples are collected along traverses in terms of water depth. The samples may be collected at increasing depth increments of two feet, for example, to a total depth of 30 or more feet. An alternative method, apparently not much used by engineers, is to space the underwater samples according to distance from the mean tide line, regardless of the local water depth.

If comparison of underwater samples are to be made on the basis of equivalent water depth, some control of this factor is desirable. On the other hand, if it is true that the nearshore bottom displays zones similar to backshore, foreshore, etc., it may be that sampling within these zones will provide equally useful data. The writer is not aware that "population zones" on the nearshore bottom have been studied as such, although data are available from maps that show the nearshore distribution of average particle sizes, degrees of sorting, etc. Inman (1953) provides a number of such maps, and recent analysis of his data by Miller (1954) shows that certain areas can be distinguished on the basis of multi-variate analysis of mean size, degree of sorting, and skewness.

The writer has observed that on some Lake Michigan beaches there is a marked coarsening of material from near the mean water line to the plunge zone. Beyond this the bottom is sometimes homogeneous to moderate



depths. Maps by Fisher (1954) show rough linearity of underwater zones, sometimes at an angle to the shore line. No "break" as conspicuous as the berm for separating the underwater populations is present.

The problem of sample spacing arises frequently in beach sampling. How far apart should samples be spaced along a profile? How far apart should the profiles themselves be spaced along the beach? Multilevel sampling, mentioned earlier, provides one way of attacking the problem.

Critical data on beach particle size are not available as an illustration, but some beach firmness data obtained at Ocean Beach, Maryland for another purpose in 1953 may be used. A portion of the backshore is arranged into six 100-foot squares as shown in Figure 5. Each of these is divided into nine 33-foot squares. Two of the 33-foot squares were selected at random in the 100-foot squares, and within the selected 33-foot squares two randomized penetrometer readings were made. The two values in each randomized 33-foot square are indicated in Figure 5. Readings were made with a dial penetrometer at 6-inch depth.

If the experiment had been designed for this multilevel study, the 100-foot squares would be spaced randomly over the backshore. The number of smaller squares within the 100-foot squares would be increased to avoid necessity for finite population corrections. The example will be treated as though it had been properly designed, in order to show the kind of data that may be obtained from such a study.

Preliminary analysis of the large squares as six groups of four items each in a single factor design similar to Table 2 indicated that the backshore in the six large squares is homogeneous in beach firmness.

For present purposes the variance components of the three sampling levels are important. These were found by use of a components of variance model for multilevel sampling (Cochran, 1953, chapter 10; Anderson and Bancroft, 1952, p. 325; Olson and Potter, 1954). The values were obtained as follows:

<u>Source</u>	<u>Variance Component</u>
Between 100-foot squares	44.3
Between 33-foot squares within 100-foot squares	43.9
Between samples within 33-foot squares	226.0

The largest variability occurs between individual samples, whereas the variance contributions of the 33-foot and 100-foot squares are roughly the same. This order need not be general, so that the following interpretations are specific instances rather than generalizations. Finite population corrections are ignored, as stated earlier.

In evaluating the penetrometer data in terms of the relative effects of sample spacing, it will in general be true that the samples within the

[illegible]

33-foot squares will be closer together than the squares themselves are in the largest units. In the present instance one may say that the variability between the closely spaced samples within a 33-foot square is greater than that between 33-foot squares or between 100-foot squares. If one wishes to obtain the best estimate of average penetrability in the backshore, the question may be raised whether it is better to take more readings close together, or more readings spread over the larger sampling units.

It can be shown (Anderson and Bancroft, 1952, p. 326) that if the number of 100-foot squares is  $a$ , the number of selected 33-foot squares within one 100-foot square is  $b$ , and if the number of samples within each 33-foot square is  $c$ , then the variance of the grand mean,  $V(\bar{\bar{X}})$ , is

$$V(\bar{\bar{X}}) = (44.3/a) + (43.9/ab) + (226.0/abc)$$

where the numbers in the numerators are taken from the preceding components of variance table. In the present experiment,  $a = 6$ ,  $b = 2$ ,  $c = 2$ . Hence,

$$V(\bar{\bar{X}}) = 7.38 + 3.66 + 9.42 = 20.46.$$

The variance of the mean is a measure of the variability to which the sample mean is subject. By using the figure 20.46, it is possible to see how alternative sampling plans would decrease this value. If the number of samples is to be doubled, this could be done by applying the same plan to twice as many 100-foot squares on the backshore, or by taking twice as many readings in the present set of 33-foot squares. In the first alternative,  $a = 12$ ,  $b = 2$  and  $c = 2$ . In the second alternative  $a = 6$ ,  $b = 2$ , and  $c = 4$ .

By inserting these values in the expression for  $V(\bar{\bar{X}})$ , the first alternative yields:

$$V(\bar{\bar{X}}) = (44.3/12) + (43.9/24) + (226.0/48) = 10.23$$

The second alternative yields:

$$V(\bar{\bar{X}}) = (44.3/6) + (43.9/12) + (226.0/48) = 15.75.$$

It is evident from these values that doubling the number of largest sampling units will cut the variance of the mean in half, whereas by doubling the number of lowest-level sampling units the variance is only decreased about 25 per cent.

If generalizations are permitted from this experiment, it would appear that a better estimate of the mean arises from taking additional samples over the larger units than by increasing the number of closely spaced samples in the smaller units. This conclusion was also reached by Potter and Olson (1954) in their study of cross-bedding of Pennsylvanian

sandstones.

Although this example from multilevel sampling does not completely answer the question of sample spacing, it does indicate that statistical methods are available for attacking these problems. Some informal experiments by the writer, based largely on student problems, suggested that no significant differences in average particle size arose from grid sampling on 15-foot and 30-foot spacings. These distances may be of the same order of magnitude in a sampling sense, however, and it may be that greater differences in relative spacing would give more conclusive results.

#### TENTATIVE PROPOSALS FOR BEACH SAMPLING

The preceding sections of this report indicate some of the complexities involved in beach sampling. It does not appear possible at present to propose a single unified sampling plan which can take account of all these complicating factors. It is possible, however, to suggest certain procedures for beach sampling which increase the reliability of the data and greatly simplify their statistical treatment. The following recommendations, qualified where necessary, represent the writer's present judgment on problems of beach sampling:

(1) Sample Size and Depth. Samples should be of the same size (volume or weight) as far as possible for any given study. This eliminates the necessity of weighting means and permits direct comparison of sets of data. Sample size involves two aspects: cross-sectional area of the sample and depth of penetration.

a. The studies of sample size given earlier indicate that for sand samples a circular area between about 1.5 and 4 inches in diameter is satisfactory. A variety of cylindrical cardboard cartons in this range is available. Ice cream cartons appear to be particularly suitable. For gravel the sample area probably needs enlargement if depth of penetration is to remain the same for both sand and gravel. On gravel beaches the samples may be taken to greater depths to compensate for area; judgment rather than fixed rules is indicated at this stage.

b. For most sand beaches a sample about 2 or 3 inches in depth seems to be satisfactory if surficial features are to be emphasized. If the upper half-inch surface layer is first scraped off, this should be done for all samples. In special studies, where individual sedimentation units are to be investigated, the methods discussed by Otto (1938) are applicable. For studies involving a greater depth of penetration some standard methods of extracting the total sample and of splitting it to size should be adopted. One way is to dig out a cylinder of sand, say about 6 inches in diameter and a foot deep, spread the sample on a tarpaulin, mix thoroughly, and quarter successively until a field sample of about 1 pint is obtained. This represents a form of subsampling, and for fundamental studies the statistical effects of such subsampling should be evaluated.

It may be pointed out that current sampling practices yield very large samples in a statistical sense. One cubic centimeter of beach sand has about 60,000 grains, so that the usual half-pint carton contains about 3 or 4 million sand grains, and weighs about 500 grams. It is likely that very much smaller samples could be used, especially for microscopic methods of particle size analysis.

(2) Randomization. A randomizing element should be made an explicit part of all sampling procedures. Tables of random numbers are generally most satisfactory for this purpose, although coin tossing may be used to decide between alternatives. As was pointed out the randomizing element eliminates personal bias and provides a basis for applications of sampling theory. For most studies it is possible to prepare the randomized locations in the office, so that the actual field sampling proceeds according to plan. Randomization of underwater samples is probably most conveniently arranged on a systematic plan along profiles, to avoid complicated maneuvering of the ship.

(3) Sample Stratification. For most beach studies a plan of stratified sampling is probably preferable to simple random sampling. Rough stratification may be accomplished by walking over the beach and noting the presence of berms, of firm and soft areas, of gravel patches or other size irregularities, of magnetite or other placer streaks, and so on. These observations may be used for dividing the beach into sampling zones. An alternative is to adopt standard procedures based on a backshore zone, upper and lower foreshore zones, and up to several nearshore bottom zones. Each zone is then sampled separately.

Underwater zones are more difficult to distinguish without some preliminary sampling, so that the usual procedures of sampling by water depth, or a modification based on systematic samples outward from mean tide level, seem adequate in the present state of knowledge.

(4) Sample Spacing and Number of Samples. Decisions on these two aspects of beach sampling are among the most important in the study. Considerations of cost, plus the fact that data from beach samples constitute only a small part of the total observations made on beaches for engineering studies, require that as few as possible samples be taken. However, certain minimum standards are dictated by good statistical practice. The individual worker has to make a decision regarding the number of safeguards he wishes to include in his sampling plan. The following considerations may be taken into account:

a. Particle size and mineral data (as well as such measurements as proportion of acid solubles) are based on large samples in a statistical sense, and hence each sample value has a reasonably high degree of reliability, assuming a standard method of analysis. In one sense, therefore, a single random sample from each beach zone along a profile, (or a set of systematic stratified samples) may afford an estimate of the zone characteristics adequate for many studies.



b. Although each particle size analysis provides an estimate of average particle size, of spread (sorting), and of skewness, these values do vary within the zones, and a better estimate is obtained by taking more than a single sample in each. It is the writer's opinion that where time and cost permit, a minimum of four samples from each beach zone may be collected in order to have a basis for estimating the variability of means within the zones. Moreover, if comparisons are to be made between different zones along a profile, or between the same zones on different profiles, sets of four samples permit convenient applications of analysis of variance.

c. Some beach observations represent single values rather than averages. Such are penetrometer readings, for example, in contrast to particle size analysis. In general these single values will fluctuate more widely than average values. If penetrometer readings are to be taken along a profile, it may be advantageous to take several readings at random in the vicinity of each sand sample rather than a single reading. This will provide a cluster sample. Another advantage of averages is that they tend to be normally distributed even when the single observations form a skewed distribution. Beach characteristics listed in Table 1 may in many cases be treated by normal approximations if averages based on a reasonable number of observations are used instead of single observations.

d. Applications of subsampling (multilevel sampling) to beach problems deserve much more attention than has been accorded them. Estimates of gross beach characteristics for extensive areas on the beach can be evaluated more thoroughly when the variability introduced by each level of sampling is included in the study. It may be found in some instances that considerable work can be saved for a given degree of reliability by using multilevel sampling methods instead of detailed sampling at the lowest level.

e. The spacing of samples along a profile has been touched upon. If the zones are sampled separately, the number of samples from each could be kept constant regardless of zone width. As an alternative, the total number of samples along the profile could be allocated according to the relative widths of the beach zones. Each of these methods carries certain statistical implications in variance computations, and the choice of method may depend upon whether an average of the entire profile is to be estimated, or whether subsequent analysis is carried along the beach zone by zone.

A related question is the spacing of profiles along the beach. The known lesser variability along than across beaches suggests that profiles may be spaced further apart along the beach than individual samples are spaced along a profile. Perhaps some general rule, such as a spacing equivalent to three times the average profile length may be applicable. In order to have sufficient data for reliable estimates of beach properties, some minimum number of profiles, such as six, should be taken. For more detailed study of local populations the principle of plane sampling discussed earlier may be applied.

f. The number of samples necessary to test hypotheses concerning differences among mean values depends upon the magnitude of the differences that it is desired to detect. The variance of the sampling distribution of the mean,  $\bar{X}$ , is  $\sigma^2/N$ , where  $\sigma^2$  is the population variance, and  $N$  is the number of samples. The variance can be reduced either by increasing the number of samples or by decreasing  $\sigma^2$ . The latter may be accomplished by increasing the precision of the methods of analysis. In practice some combination of these possibilities may be used. Dixon and Massey (1951, p. 121, 219-220) discuss methods that may be used.

(5) Analysis of Samples. The same methods of laboratory analysis should be used throughout any one study. This assures that the numbers used in the statistical analysis have the same operational definition. That is, the process of acquiring the numbers is kept constant, so that no variability is introduced into the data beyond that inherent in any single method of laboratory analysis. If operator judgment is required in the analysis, a single analyst should be used, or provisions made for evaluating operator effects. (See Griffiths and Rosenfeld, 1954, on this point).

Statistical analysis is facilitated when measurements are normally distributed. This may be checked by arranging a cumulative distribution as in particle size analysis, and plotting the points on arithmetic probability paper. In analysis of variance the data may depart from normalcy, although the variance of the groups should be independent of the group means. This can be checked by a scatter diagram. If sufficient replication is available, the variance of the groups should also be checked for homogeneity (Dixon and Massey, 1951, p. 147). In some instances (as with counted data) transformations are appropriate (Bartlett, 1947, and Kempthorne, 1953, p. 153). Table 1 may be checked for suggestions on distributions.

As a general rule it is better to analyze each beach sample separately and to combine the data algebraically, than to mix the samples before analysis. This separate treatment provides data for estimates of variability between samples, which is lost if only the composite data are available.

(6) Repetitive Sampling. Samples collected on a given date represent only one of a large number of possible conditions which may occur on beaches. If an estimate is to be made of the average composition of a beach during the entire year, samples will be required during the several seasons as well as during quiet and stormy conditions. It is not difficult to obtain such data on the exposed part of the beach, but the collection of underwater samples may not be feasible during severe storms.

Repetitive sampling may be designed in several ways to obtain a set of samples representative of the range of seasonal and storm cycle conditions on a beach. Even without specific seasonal design, sets of four samples from each zone collected at intervals through the year provide data for comparisons by analysis of variance and other techniques. It is probably preferable to randomize the specific sampling positions along the profiles separately for each sampling date.

Problems of weighting the data may be involved in undesigned repetitive sampling, unless some provision is made to prorate the number of sampling intervals according to seasons and storms. For example, if 15 sets of samples are taken on quiet days and only one set on a stormy day, the mean value may not be characteristic of the average beach conditions. Stratification may be applied to the problem to overcome some of these difficulties. Two possibilities appear to apply:

a. The year may be stratified into months or seasons, with samples collected systematically or at random in each time stratum. Suppose the year is stratified into 12 months, with two sets of samples to be collected from each zone each month. The sampling dates in each month may then be separately randomized, or the dates may be randomized for the first month, and repeated as systematic stratified sampling dates in the remaining months. An alternative here is to stratify according to four 3-month seasons, with six randomized or systematized sampling dates in each season.

b. Instead of stratifying the year according to months or seasons, the total year may be divided into several strata representing quiet intervals, mild storms, severe storms, etc., according to data available on the relative proportions of time that each set of conditions may be expected to occur. Then by distributing the 24 sets of samples over these strata in random or systematic manner, the sample data will be representative of physical conditions rather than simply of time.

Stratification of the year into time periods or into strata defined by occurrence of given processes involves problems of cyclical phenomena. Complete randomization within the time strata may yield more representative samples than systematization within strata, if by chance the systematic points fall at corresponding highs or lows in the cycle. Cochran (1953) discusses some of the factors involved, and it is likely that collaboration of mathematical statisticians will be required to solve the problem of reliable repetitive sampling.

#### CONCLUDING REMARKS

The discussion of beach sampling in this report is given from a subject-matter viewpoint rather than from a theoretical statistical viewpoint. The recommended sampling procedures are the writer's interpretation of sampling principles and methods, based largely on Cochran's (1953) classification, and on a number of beach sampling experiments at Northwestern University, many conducted by graduate students.

The writer has no misconceptions about the tentative nature of the sampling methods suggested in this report. The problem of beach sampling requires the active collaboration of mathematical statisticians, but the statistician can be of little help if poorly collected data are submitted to him and he is asked what can be done with them. Sampling design is necessary if rigorous analysis of the results is to be had. It is the



writer's hope that by including more explicit design elements in future sampling programs, material will be made available by engineers and geologists for further analysis by statisticians. From the results of such analysis, modified where needed by practical considerations, valid sampling programs applicable to specific beach problems can be developed.

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